

AD-A129 674

TECHNIQUES FOR MODELING STOCHASTICS DYNAMICAL SYSTEMS
(U) HARVARD UNIV CAMBRIDGE MA DIV OF APPLIED SCIENCES
R W BROCKETT 26 MAY 83 ARO-13605.14-MA DAAG29-79-C-0147

1/1

UNCLASSIFIED

F/G 12/1

NL



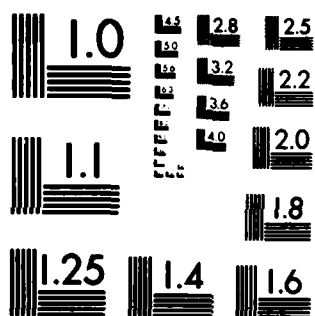
END

DATE

FILED

1-7 83

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

ARO 13605.14-MA

16465.3-MA

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|--|-----------------------|--|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| | AD-A129674 | |
| 4. TITLE (and Subtitle) | | 5. TYPE OF REPORT & PERIOD COVERED |
| Techniques for Modeling Stochastics Dynamical Systems | | Final Report 4 Feb. 1976 - 1 Jan. 1983 |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| | | |
| AUTHOR(s) | | 8. CONTRACT OR GRANT NUMBER(s) |
| R. W. Brockett | | Grant DAAG29-76-G-0139 Grant DAAG29-79-C-0147 |
| PERFORMING ORGANIZATION NAME AND ADDRESS | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| Division of Applied Sciences Harvard University | | |
| CONTROLLING OFFICE NAME AND ADDRESS | | 12. REPORT DATE |
| U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709 | | May 26, 1983 |
| MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 13. NUMBER OF PAGES |
| U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709 | | 7 |
| | | 15. SECURITY CLASS. (of this report) |
| | | Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 6. DISTRIBUTION STATEMENT (of this Report) | | |
| Approved for public release; distribution unlimited. | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| <div style="text-align: right;"> DTIC SELECTE JUN 23 1983 A </div> | | |
| 18. SUPPLEMENTARY NOTES | | |
| The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) | | |
| Stochastic modelling, estimation, nonlinear systems, Wiener processes, white noise, Poisson processes. | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) | | |
| This report summarizes the principle research findings of 30 published papers and 3 Ph.D. theses completed with partial support of the Army Research Office. | | |

83 06 22 034

ADA 129674

DTIC FILE COPY

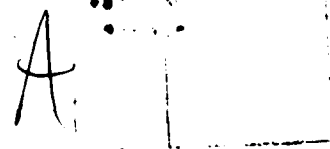
FINAL REPORT

U.S. ARMY RESEARCH GRANT/CONTRACT

DAAG29-76-G-0139
DAAG29-79-C-0147

1. Statement of the Problem Studied

Nonlinear stochastic systems arise in various areas of engineering including communications, vibrations and control. Under the grant and contract referenced above ^{the author} ~~we~~ undertook to produce rich class models which were, at the same time, analytically tractable. ^{He} ~~We~~ studied these questions with respect to computing the statistical properties of solutions and with respect to finding sufficient statistics for nonlinear estimation problems. ~~We~~ ^{were} also investigated the specific questions which arise in an attempt to adaptively control systems via system identification and subsequent control.



2. Summary of the Most Important Results

In the period covered by this final report we have studied stochastic modeling and the implications for estimation and control. This has resulted in a large number of publications. The most significant results obtained seem to be the following:

A. Our results which show how to compute all moments and all multipoint (in time) statistics associated with linear and bilinear stochastic differential equations whose coefficients are finite state jump processes or white noise. (See [5], [8], [11], and [18].) This work completely systematizes earlier ad hoc treatments of special cases and has been used by others in the study of failure detection, reliability of electrical distribution grids, etc. It defines the role of the Poisson counter in modeling finite state processes and illustrates its utility in modeling.

B. Our results which describe the connection between sufficient statistics for nonlinear estimation problems and Lie algebras. (See [13], [16], [17], and [22].) This work succeeded in bringing together two major research activities -- nonlinear filtering and nonlinear realization theory -- in order to understand in a very natural way the intrinsic complexity of a given nonlinear filtering problem. A major part of the 1980 NATO Conference on Stochastic Systems was devoted to this work and it has been taken up by others subsequently.

C. Our research which has had significant conclusions concerning the role of geometry on system identification. (See [12], [14], and [31].) This work has given rise to a new point of view on the identification of linear systems and a reexamination of the performance of self timing regulations as applied to linear systems.

D. Our results concerning the linearization of nonlinear systems by feedback. (See [7].) Here we take up the question of linearization by feedback and give a testable criterion for linearizability in terms of a certain Lie algebraic condition. This line of work has subsequently been pursued intensively by people at NASA-Ames and carried by them to the flight test stage. Our more recent paper [24] represents a first step toward making these local results global.

3. List of all Publications and Technical Reports Published

- [1] "The Lie Groups of Simple Feedback Systems," IEEE Decision and Control Conference, 1976.
- [2] "Convergence of Volterra Series on Infinite Intervals and Bilinear Approximations," Nonlinear Systems and Applications (V. Lakshmikanthan, ed.), Academic Press, pp. 39-46, 1977.
- [3] "Control Theory and Analytical Mechanics," in Geometric Control Theory (C. Martin and R. Hermann, eds.), Math. Sci. Press, Brookline, MA, pp. 1-48, 1977.
- [4] "Optimal Linear Systems with Polynomial Performance Measures," 1977 Joint Automatic Control Conference.
- [5] "Stationary Covariance Generation with Finite State Markov Processes," 1977 Joint Automatic Control Conference.
- [6] "Stochastic Bilinear Models," 1977 Joint Automatic Control Conference.
- [7] "Feedback Invariants for Nonlinear Systems," Proc. of the 1978 IFAC Congress, Helsinki, Finland.
- [8] "A Representation Theory for Linear Differential Equations with Markovian Coefficients," 1977 Allerton Conference (with G. Blankenship).
- [9] "Lie Algebras and Rational Functions: Some Control Theoretic Connections," in Lie Theories and Their Applications (W. Rossman, ed.), pp. 268-280, Dept. of Mathematics, Queen's University, Kingston, Ontario, 1978.
- [10] "On Improving the Circle Criterion," Proc. of the IEEE Decision and Control Conference, New Orleans, LA, 1977.
- [11] "Modeling and Estimation with Bilinear Stochastic Systems," Trans. of the 24th Conference of Army Mathematicians (ARO Report 79-1), pp. 395-504, 1979.
- [12] "A Scaling Theory for Linear Systems," IEEE Trans. on Automatic Control, Vol. 25, pp. 197-207, 1980 (with P.S. Krishnaprasad).
- [13] "The Geometry of the Conditional Density Equations," in Analysis and Optimization of Stochastic Systems (O.L.R. Jacobs et al, eds.), Academic Press, N.Y., pp. 299-309, 1980 (with J.M.C. Clark).

- [14] "The Geometry of the Partial Realization Problem," Proc. of the 1978 Conference on Decision and Control, IEEE, N.Y.
- [15] "Stochastic Control and the Second Law of Thermodynamics," Proc. of the 1978 Conference on Decision and Control, IEEE, N.Y. (with J.C. Willems).
- [16] "Some Remarks on Finite Dimensional Nonlinear Estimation," Asterique, Societe de Matematica de France, Vol. 75-76, pp. 47-55, 1980.
- [17] "Classification and Equivalence in Estimation Theory," 1979 Conference on Decision and Control, IEEE, N.Y., 1979.
- [18] "Stochastic Realization Theory and the Planck Law for Black Body Radiation," Ricerche di Automatica, Vol. 10, pp. 344-362, 1979.
- [19] "Polynomials, Bilinear Forms, and Representations of Lie Algebras," in Algebraic and Geometric Methods in Linear System Theory (AMS Lectures on Applied Mathematics, Vol. 18 [C.I. Byrnes and C. Martin, eds.]), pp. 1-6, 1980.
- [20] "Multivariable Nyquist Criterion, Root Locus, and Pole Placement by Output Feedback," IEEE Trans. on Automatic Control, Vol. AC-26, pp. 271-284, 1981 (with C.I. Byrnes).
- [21] "Control Theory and Singular Riemannian Geometry," in New Directions in Applied Mathematics (P. Hilton and G. Young, eds.), Springer-Verlag, pp. 11-27, 1981.
- [22] "Nonlinear Systems and Nonlinear Estimation Theory," in Stochastic Systems (M. Hazewinkel and J.C. Willems, eds.), Reidel Publishing Co., Dordrecht, The Netherlands, pp. 441-477, 1981.
- [23] "Asymptotically Optimal Estimation," Proc. of the 1981 Conference on Decision and Control, IEEE, pp. 76-79, 1981.
- [24] "The Global Description of Locally Linear Systems," Springer-Verlag Lecture Notes on Control and Information Sciences, Vol. 39 (D. Hinrichsen and A. Isidori, eds.), pp. 1-8, 1982.
- [25] "Asymptotic Stability and Feedback Stabilization," in Differential Geometrics in Control Theory (R.W. Brockett, et al, eds.), (Progress in Mathematics, Vol. 27), Birkhauser Publishers, Boston, MA, pp. 181-191, 1983.
- [26] "Linear Feedback Systems and the Groups of Lie and Galois," Linear Algebra and Its Applications, Vol. 24, (1983).

- [27] W.S. Wong, "Volterra Series, Universal Bilinear Systems and Fock Representations," Proc. of the Thirteenth Annual Conf. on Information Sciences and Systems, Johns Hopkins University, 1979.
- [28] P.S. Krishnaprasad, "Geometry of Parametric Models -- Some Probabilistic Questions," Proc. of the 15th Allerton Conf. on Control, Communicating and Computing, 1977, pp. 661-670.
- [29] M.A. Shayman, "Geometry of the Algebraic Riccati Equation, Part I," SIAM J. Control and Optimization, Vol. 21, No. 3, May 1983, pp. 375-394.
- [30] M.A. Shayman, "On the Variety of Invariant Subspaces of a Finite-Dimensional Linear Operator," Transactions of the American Mathematical Society, Vol. 274, No. 2, December 1982, pp. 721-747.

4a. List of All Participating Scientific Personnel

Roger W. Brockett
 Anthony Bloch
 Varda Haimo
 P.S. Krishnaprasad
 Josip Lončarić
 Alan McIvor
 Mark Shayman
 Tom Taylor
 Wing Wong

4b. List of All Participating Scientific Personnel Showing Any Advanced Degrees Earned by Them While Employed on the Project

- [31] P.S. Krishnaprasad, "Geometry of Minimal Systems and the Identification Problem," Ph.D. Thesis, Harvard University, August 1977.
- [32] W.S. Song, "Operator Theoretic Methods in Nonlinear Systems," Ph.D. Thesis, Harvard University, August 1980.
- [33] M.A. Shayman, "Varieties of Invariant Subspaces and the Algebraic Riccati Equation," Ph.D. Thesis, Harvard University, December 1980.